AN EQUILIBRIUM MODEL OF MANAGERIAL COMPENSATION

Michael MAGILL

Department of Economics
University of Southern California
Los Angeles, CA 90089-0253
magill@usc.edu

Martine QUINZII

Department of Economics
University of California
Davis, CA 95616-8578
mmquinzii@ucdavis.edu

March 2005

Abstract. This paper studies a general equilibrium model with two groups of agents, investors (shareholders) and managers of firms, in which managerial effort is not observable and influences the probabilities of firms' outcomes. Shareholders of each firm offer the manager an incentive contract which maximizes the firm's market value, under the assumption that the financial markets are complete relative to the possible outcomes of the firms. The paper studies two sources of inefficiency of equilibrium. First, when investors are risk averse and effort influences probability, market-value maximization differs from maximization of expected utility. Second, because the optimal contract exploits all sources of information for inferring managerial effort, when firms' outputs are correlated the contract of a manager depends on the outcomes of other firms. This leads to an external effect of the effort of one manager on the compensation of other managers, which market-value maximization ignores. We show that under typical conditions these two effects lead to an underprovision of effort in equilibrium. These inefficiencies disappear however if each firm is replicated, and in the limit there is a continuum of firms of each type.

An Equilibrium Model of Managerial Compensation

Introduction

In the 1990's executive compensation evolved towards high-powered incentive contracts designed to align the incentives of CEOs with the interests of their firms' shareholders.[1] The economic model which has come to serve as the basis for discussing incentive contracts and executive compensation is the principal-agent model. However the CEO of a large corporation runs the firm not for a single principal but for many principals, namely all the shareholders of the firm. To avoid the difficulties of a model with multi-principals it is typically assumed that firms' shareholders are risk neutral, so that they all agree to choose contracts which maximize firms' net expected profits. However the high equity premium observed on the stock market shows that risk neutrality of shareholders is not a realistic assumption, and it can not be adopted in a model that studies the relation between executive compensation and prices on the financial markets.

The second element which is missing in a discussion of CEO compensation in the context of a bilateral principal-agent model, is the important stylized fact that firms' profits are typically strongly correlated. This suggests that firms' outcomes are affected by common factors and that the optimal contract of a CEO should take this into account by incorporating some element of relative performance.

In this paper we present a model which combines the determination of executive compensation studied in the principal-agent model with an equilibrium model of financial markets, and which permits these two stylized facts—risk aversion of the shareholders and correlation of firms' outcomes—to be taken into account. The model consists of a two-period economy with two groups of agents, I investors (or shareholders) and K managers of K firms, in which managerial effort is not observable and influences the probabilities of the firms' outcomes. To simplify the model the assignment of managers to firms is taken as given. At date 0 there is trade on the financial markets and the shareholders (or the Board of Directors) of each firm offer an incentive contract to the firm's manager. We make two additional simplifying assumptions: first the financial markets are complete relative to the possible outcomes of firms, and second, managers cannot undo the incentive contracts they are offered by trading on the financial markets. The hypothesis of complete financial markets per-

[1]The evolution is clear when comparing Murphy's recent assessment of CEO compensation (Murphy (2001)) with that in Jensen-Murphy (1990).

mits optimal contracts to be defined in the presence of many principals: all the shareholders of a firm agree with the objective of choosing the manager's contract to maximize the present value of its profit or, equivalently, the market value of the firm.

The object of this paper is to study the normative properties of the equilibria of this model, which embeds a family of principal-agent models (one for each firm) into a model of financial market equilibrium. Does the choice of contracts which maximize market value lead to an optimal choice of effort for the firms' managers? We find that the conditions under which market-value maximization leads to constrained Pareto optimality are restrictive: investors must be risk neutral and firms' outcomes must be independent. Thus under the assumptions which best reflect the stylized facts about equity markets—risk-averse investors and correlated outcomes of firms—the equilibrium levels of managerial effort are not socially optimal.

To clarify the sources of the inefficiency we decompose the study of the model into two special cases. In the first, investors are risk averse but we retain the assumption of independence of firms' outcomes; in the second, firms' outcomes are correlated but investors are taken to be risk neutral. There are several ways of modeling the correlation of firms' outcomes: here we assume that firms are affected by a common shock. This gives a relatively precise structure to the model, and in keeping with the key hypothesis of the principal-agent model, that idiosyncratic shocks are unobservable, we make the assumption that the common shock is not observable either.

The first source of inefficiency, linked to the risk aversion of the shareholders, comes from the structural property of the principal-agent model, that managerial effort affects the probabilities of the firms' outcomes. When investors trade on the financial markets they evaluate the probabilities of outcomes—correctly under the assumption of rational expectations—and this evaluation influences the security prices. But effort shifts probabilities, and security prices do not provide a signal of the value of shifting probabilities. Rather they provide a well-defined value for income in each state, expressed by the stochastic discount factor that is used by the firms to maximize profit. We show that under these circumstances maximizing a weighted sum of expected utilities of the investors (what a planner does) and maximizing the present value of the firms' profits (what the equilibrium does) give different results, the equilibrium leading to an underprovision of effort.

The second source of inefficiency, linked to the correlation of firms' outcomes, is perhaps more intuitive since it induces an externality between the actions of the firms' managers—and markets typically fail to take externalities into account. An optimal contract rewards a manager in circumstances which are more likely to occur with high effort and penalizes the manager in circumstances which are more likely with low effort. All available sources of information are used to infer the likelihood of effort for a given outcome. In the presence of a common shock, the realized outcomes of other firms provide information on the value of the common shock and hence by inference on the likelihood of the manager's effort. However since the outcomes of the other firms are in turn influenced by the effort levels of their managers, the use of information creates an externality of the effort of one manager on the expected utility of other managers. We show that the externality is quite subtle and can be decomposed into two separate effects arising from an increase in effort on the part of a manager. The first, which we call the *direct effect*, is to decrease the compensation of other managers: this is akin to a standard negative externality. The second, which we call the *information effect* is to change the likelihood ratios of the managers of other firms thereby changing the information that is used to deduce the effort of the managers from the outcomes of their firms: this is akin to a positive externality. We show that if the correlation of firms' outcomes is sufficiently strong, then the information effect tends to dominate, leading to an underprovision of effort in equilibrium.

Thus market value maximization does not lead to a socially optimal choice of managerial effort. However the inefficiencies may be small: in particular, if the individual firms are replicated and in the limit replaced by a continuum of identical firms with identical managers and independent (or conditionally independent) outcomes, then the inefficiencies disappear. The result of constrained optimality of equilibrium, obtained in insurance models with a continuum of agents of each type, also obtains here. Nevertheless, if the assumption of a continuum of firms is convenient for cancelling the inefficiencies which are the subject of this paper, it is a much less natural assumption to make for analyzing the contracts of CEOs of large corporations than the assumption of a continuum of identical agents in a model of insurance under moral hazard.

1. The model

Consider a one-good, two-period economy in which there are two groups of agents, I investors and K managers, and a collection of K firms, each run by one of the managers. The match between managers and firms is taken as given, so that the only option that remains is that manager k runs firm k or takes an outside option yielding a utility level ν_k. Uncertainty is described by a finite number of possible outcomes for each firm at date 1: for firm k the outcomes are $y^k_{s_k}$, $s_k \in S_k$, indexed in increasing order, that is $s_k > s'_k$ implies $y^k_{s_k} > y^k_{s'_k}$. A state of the economy at date 1 is a K-uple $s = (s_1, \ldots, s_K)$ describing the the realized output (or profit) of each firm: we let $S = S_1 \times \ldots \times S_K$ denote the state space, $y_s = (y^1_{s_1}, \ldots, y^K_{s_K})$, $s \in S$, denoting the vector of outputs of the K firms in state s.

To study an equilibrium model of managerial compensation in the presence of moral hazard we assume that the probabilities of the possible outcomes of firm k are influenced by the entrepreneurial input (effort for short) of its manager, $e_k \in \mathbb{R}_+$, which is assumed to be unobservable. To permit common as well as idiosyncratic shocks to influence the outcomes of the firms let $p(s, e) = p(s_1, \ldots, s_K, e_1, \ldots, e_K)$ denote the joint probability of the outcomes, given the effort levels $e = (e_1, \ldots, e_K)$ chosen by the managers. The function p is assumed to be common knowledge for the agents in the economy. When we need to focus on the (representative) firm k, it will be convenient to use the notation $s = (s_k, s^{-k})$ and $e = (e_k, e^{-k})$, where $s^{-k} = (s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K)$ and e^{-k} is defined in the same way.

All agents in the economy, investors and managers, are assumed to have expected-utility preferences over date 1 consumption streams—at date 0 agents make trades on financial markets and write contracts, but there is no date 0 consumption. Let u_i denote the VNM utility index of investor i, $i \in I$, and v_k that of manager k, $k \in K$. The disutility of effort for manager k is assumed to enter additively and is expressed by a cost function $c_k(e_k)$.

Each firm is owned by a subset of investors with ownership shares δ^i_k, $i \in I$, $k \in K$, δ^i_k denoting the share of investor i in firm k. While we have in mind that investors trade on equity markets and hedge their risks by using options and derivative markets, it will simplify the analysis to assume an equivalent complete financial market structure consisting of a complete set of Arrow securities, security s promising to deliver 1 unit of good (income) in state s and nothing otherwise. These securities are traded at date 0, with $\hat{\pi}_s$ denoting the price of security s.

At date 0, the shareholders of firm k choose the contract $\tau^k = (\tau^k_s, s \in S)$ that they propose to their manager. Since manager k acts as an agent for several principals, we cannot assume as in the standard principal-agent model that each shareholder chooses the contract that maximizes his/her utility. On the other hand since markets are complete with respect to outcomes, the present value of the firm's profit $\sum_{s\in S} \hat{\pi}_s(y^k_s - \tau^k_s)$ is well defined and all shareholders will agree with the objective of choosing the contract for manager k which maximizes the present value of the firm's profit (its market value). We are thus led to the following concept of equilibrium.

Definition 1. A *managerial equilibrium* is a pair of actions and prices $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi}) = ((\bar{x}^i)_{i\in I}, (\bar{\tau}^k, \bar{e}^k)_{k\in K},\ \bar{\pi})$ consisting of consumption streams for investors, contracts and effort levels for managers, and prices, such that

(i) for $k \in K$, shareholders of firm k choose $(\bar{\tau}^k, \bar{e}^k)$, the contract of the manager and the effort level to induce, which maximizes the market value of the firm:

$$\sum_{s\in S}(y_s^k-\tau_s^k)\bar{\pi}_s p(s,e^k,\bar{e}^{-k})$$

on the set of $(\tau^k,e^k)\in \mathbb{R}_+^S\times\mathbb{R}_+$ satisfying

$$\sum_{s\in S} v_k(\tau_s^k)p(s,e^k,\bar{e}^{-k})-c_k(e_k)\geq \nu_k \tag{PC$_k$}$$

$$e_k\in \operatorname{argmax}\left\{\sum_{s\in S} v_k(\tau_s^k)p(s,e^k,\bar{e}^{-k})-c_k(e_k)\mid e_k\in\mathbb{R}_+\right\} \tag{IC$_k$}$$

(ii) for $i\in I$, investor i chooses the optimal consumption stream

$$\bar{x}^i\in \operatorname{argmax}\left\{\sum_{s\in S}u_i(x_s^i)p(s,\bar{e})\right\}$$

subject to the (present-value) budget constraint

$$\sum_{s\in S}\hat{\pi}_s x_s^i\leq \sum_{k\in K}\delta_k^i\sum_{s\in S}\hat{\pi}_s(y_s^k-\bar{\tau}_s^k)$$

where

(iii) $\hat{\pi}_s=\bar{\pi}_s p(s,\bar{e}),\quad s\in S$

(iv) markets clear: $\sum_{i\in I}\bar{x}_s^i+\sum_{k\in K}\bar{\tau}_s^k=\sum_{k\in K}y_s^k,\quad s\in S$

The same definition without the incentive constraints (IC$_k$) defines a *managerial equilibrium with observable effort*, which is a useful reference concept for clarifying the effect of moral hazard. If all the agents' consumption streams are positive and all managers exert positive effort levels in the equilibrium, we will say that the equilibrium is *interior*.

Remark 1: (i) expresses the profit maximization of the firms, which defines the interaction between markets and contracts: the shareholders of each firm (or its Board of Directors) choose the contract which maximizes the present value of its profit, subject to the usual participation and incentive constraints. Of course this is a simplified view of the way executive compensation is chosen, for it assumes that the interests of the shareholders are perfectly taken into account by the Board of Directors. It also makes the simplifying assumption that the firms' managers do not have access to financial markets, so that the managers' consumption streams coincide with their compensation: $x^k=\tau^k$, $k\in K$. In some cases, for example when the firms' outcomes are independent, this is not too restrictive an assumption since the optimal contracts induce optimal risk sharing subject to the incentive constraint, and would not change if managers could trade all securities except those related to the firm they manage. However the model factors out the possibility that managers have other sources of income which change the share of their income which is "at risk" in case of a bad outcome for their firm, and hence alter the incentive properties of the compensation offered by the contracts.

We do not model the market for managers, and the optimal matching of managers to firms: instead we assume that the optimal matching has be done and, in the spirit of the principal-agent literature, we take as exogenous the expected utility that managers can obtain in their second best options. The resulting model is the simplest extension of the principal-agent model to a general equilibrium setting which permits us to study whether contracts determined by profit maximization lead to a second-best optimum. To maximize profit, or equivalently the market value of the firm, the shareholders must anticipate how the manager's effort influences the market value of the firm. The expression for the profit in (i) and the relation (iii) between $\bar{\pi}_s$ and the Arrow-Debreu prices $\hat{\pi}_s$ combine rational expectations and a competitive assumption. In equilibrium agents correctly anticipate the effort levels $\bar{e}$ of the managers, and hence the probability $p(s, \bar{e})$ of the different outcomes. From this they can deduce $\bar{\pi}_s$, the stochastic discount factor for state s, which is constant with risk neutrality and varies with the aggregate output with risk aversion. This is the rational expectations part. In the calculation of the market value of the firm in (i), $\bar{\pi}_s$ as well as the effort of the other managers, $\bar{e}^{-k}$, are taken as given, and only the effect of the manager 's effort e_k on the probability $p(s, e^k, \bar{e}^{-k})$ is taken into account. This is the competitive part of the anticipations assumption.

To study the normative properties of a managerial equilibrium we will compare it with the allocation that would be chosen by a planner seeking to maximize social welfare subject to the same incentive constraints as those faced by the firms' shareholders.

Definition 2. An allocation $(x, \tau, e) = ((x^i)_{i\in I}, (\tau^k, e_k)_{k\in K})$ is *constrained feasible* if

$$\textstyle\sum_{i\in I} x^i_s + \sum_{k\in K} \tau^k_s = \sum_{k\in K} y^k_s, \quad s \in S \tag{RC$_s$}$$

and if for all $k \in K$ the effort level e_k is optimal given τ^k, e^{-k}, i.e

$$e_k \in \operatorname{argmax}\left\{\sum_{s\in S} v_k(\tau^k_s) p(s, e^k, \bar{e}^{-k}) - c_k(e_k) \mid e_k \in \mathbb{R}_+\right\} \tag{IC$_k$}$$

An allocation (x, τ, e) is *constrained Pareto optimal* (CPO) if it is constrained feasible and there does not exist another constrained feasible pair which is weakly preferred by all agents, and strictly by at least one agent. The same definition without the incentive constraints (IC_k) defines a *first best optimum.*

First-order conditions. A natural approach to comparing equilibrium allocations $(\bar{x}, \bar{\tau}, \bar{e})$ with constrained Pareto optimal allocations is to compare the first-order conditions (FOCs) for equilibrium and constrained optimality. To derive the FOCs, consider a setting in which the incentive constraint (IC_k) can be replaced by the first-order condition for optimality of effort e_k

$$\sum_{s \in S} v_k(\tau_s^k) \frac{\partial p(s,e)}{\partial e_k} - c_k'(e_k) = 0 \tag{IC'_k}$$

Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be an interior equilibrium. To simplify notation[2] set $p(s,e) = p_s$. Then there exists a vector of multipliers $(\bar{\lambda}, \bar{\beta}, \bar{\mu}) = ((\bar{\lambda}_i)_{i \in I}, (\bar{\beta}_k, \bar{\mu}_k)_{k \in K}) \geq 0$ such that

$$\begin{aligned}
&\text{(i)} \quad u_i'(\bar{x}_s^i) = \bar{\lambda}_i \bar{\pi}_s, \quad s \in S,\ i \in I \\
&\text{(ii)} \quad \left(\bar{\beta}_k + \bar{\mu}_k \frac{\frac{\partial p_s}{\partial e_k}}{p_s} \right) v_k'(\bar{\tau}_s^k) = \bar{\pi}_s, \quad s \in S,\ k \in K \\
&\text{(iii)} \quad \sum_{s \in S} \bar{\pi}_s (y_s^k - \bar{\tau}_s^k) \frac{\partial p_s}{\partial e_k} + \bar{\beta}_k \left(\sum_{s \in S} v_k(\bar{\tau}_s^k) \frac{\partial p_s}{\partial e_k} - c_k'(\bar{e}_k) \right) + \\
&\qquad \bar{\mu}_k \left(\sum_{s \in S} v_k(\bar{\tau}_s^k) \frac{\partial^2 p_s}{(\partial e_k)^2} - c_k''(\bar{e}_k) \right) = 0,\ k \in K
\end{aligned} \tag{FOC_E}$$

where $\bar{\lambda}_i$ is the multiplier associated with the budget constraint in investor i's utility maximization problem, and $(\bar{\beta}_k, \bar{\mu}_k)$ are the multipliers associated with the participation constraint (PC_k) and the transformed incentive constraint (IC'_k) for manager k. If effort is observable, the incentive constraints do not exist (are not binding) and the FOCs are the same with $\bar{\mu} = 0$. If effort is unobservable and (IC'_k) is binding, the second term in (iii) is equal to zero.

If (x, τ, e) is an interior constrained Pareto optimal allocation then for some positive weights $(\alpha, \beta) \in \mathbb{R}_+^{I+K}$, it will maximize the social welfare function

$$W_{\alpha,\beta}(x, \tau, e) = \sum_{i \in I} \alpha^i \sum_{s \in S} u_i(x_s^i) p(s,e) + \sum_{k \in K} \beta_k \left(\sum_{s \in S} v_k(\tau_s^k) p(s, e^k, e^{-k}) - c_k(e_k) \right)$$

subject to the constraints

$$\sum_{i \in I} x_s^i + \sum_{k \in K} (\tau_s^k - y_s^k) = 0, \quad s \in S \tag{RC_s}$$

[2]Depending on the circumstances we will use the notation $p(s,e)$ or $p(s_k, s^{-k}, e_k, e^{-k})$, or, in complex expressions, the shorter notation $p_s(e)$ or even just p_s.

$$\sum_{s\in S} v_k(\tau_s^k)\frac{\partial p(s,e)}{\partial e_k} - c_k'(e_k) = 0, \quad k \in K \tag{IC$'_k$}$$

where the incentive constraints (IC_k) have been replaced by the first-order conditions (IC'_k). Thus there will exist non-negative multipliers $((\pi_s)_{s\in S}, (\mu_k)_{k\in K}$ such that

$$\begin{aligned}
&\text{(i)}^* && \alpha_i u_i'(x_s^i) = \pi_s, \quad s \in S,\ i \in I \\
&\text{(ii)}^* && \left(\beta_k + \mu_k \frac{\frac{\partial p_s}{\partial e_k}}{p_s}\right) v_k'(\tau_s^k) = \pi_s, \quad s \in S,\ k \in K \\
&\text{(iii)}^* && \sum_{i\in I}\alpha_i \sum_{s\in S} u^i(x_s^i)\frac{\partial p_s}{\partial e_k} + \sum_{j\neq k}\sum_{s\in S}\left(\beta_j\frac{\partial p_s}{\partial e_k} + \mu_j \frac{\partial^2 p_s}{\partial e_j \partial e_k}\right) v_j(\tau_s^j) \\
& && +\beta_k\left(\textstyle\sum_{s\in S} v_k(\tau_s^k)\frac{\partial p_s}{\partial e_k} - c_k'(e_k)\right) + \mu_k\left(\textstyle\sum_{s\in S} v_k(\tau_s^k)\frac{\partial^2 p_s}{(\partial e_k)^2} - c_k''(e_k)\right) = 0, k \in K
\end{aligned} \tag{FOC$_{CP}$}$$

where α_i (resp β_k) is the weight of investor i (manager k) in the social welfare function, π_s (or more accurately $\pi_s p_s$) is the multiplier associated with the resource constraint for state s, and μ_k is the multiplier associated with the incentive constraint for manager k. As before, if effort is observable $\mu = 0$, while if effort is not observable the third term in (iii)* is equal to zero.

The FOCs (i), (ii) and (i)*, (ii)* which describe how risk is distributed between investors and managers so as to induce the appropriate effort on the part of the managers are the same, implying that the contracts which are optimal from the point of view of the shareholders to induce given effort levels of the managers are also the socially efficient way of inducing this effort. The FOCs (iii) and (iii)* however are different: while they evaluate the marginal cost of an additional unit of effort by manager k in the same way, they differ in the way they evaluate its marginal benefit. For the planner, the social benefit is measured by its effect on the expected utility of all other agents in the economy, namely all investors $i \in I$ and all managers $j \in K, j \neq k$, with incentive-corrected weights, while in equilibrium the marginal benefit of manager' k effort is measured by its effect on the profit of firm k. We will show however that these apparently distinct ways of measuring marginal benefit in fact coincide when investors are risk neutral ($u_i(x^i) = x^i$) and firms' outcomes are independent. Proving this property will then suggest that in all other cases the FOCs for optimal effort (iii) in equilibrium and (iii)* in a social optimum are different.

Definition: The random outcomes of the firms are *independent* if for each $k \in K$ there exists a probability function $p_k(\cdot, e_k)$ on S_k, which depends on the effort of manager k, and

$$p(s,e) = \prod_{k\in K} p_k(s_k, e_k)$$

Since the FOCs are necessary but, because of possible non-convexities, are not always sufficient for constrained efficiency we will show that under risk-neutrality and independence a managerial equilibrium is CPO without calling on the first-order conditions.

Proposition 1. *If all investors are risk neutral and firms' outcomes are independent, if the VNM utility indices of the managers are strictly concave and satisfy $v_k(c) \to -\infty$ as $c \to 0$, , then a managerial equilibrium is constrained Pareto optimal.*

Proof. Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be a managerial equilibrium. We first show that $\bar{\tau}_k(s_k, s^{-k})$ depends only on s_k and is independent of the realizations s^{-k} of the other firms. Suppose not, i.e. suppose that for two outcome states $s = (s_k, s^{-k})$ and $s' = (s'_k, s'^{-k})$, with $s_k = s'_k$, we have $\bar{\tau}^k(s) \neq \bar{\tau}^k(s')$. For a random variable $\xi : S \to \mathbb{R}$, let $E_e(\xi) = \sum_{s \in S} p(s, e)\xi(s)$ denote its expectation given the vector e of effort levels. By the independence assumption

$$E_{\bar{e}} v_k(\bar{\tau}^k) = \sum_{s_k \in S_k} p_k(s_k, \bar{e}_k) \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k}) v_k(\bar{\tau}^k(s_k, s^{-k})) \tag{1}$$

Define $\tilde{\tau}(s_k) = \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k})(\bar{\tau}^k(s_k, s^{-k}))$. Since $\bar{\tau}^k(s) \neq \bar{\tau}^k(s')$, by strict concavity of v_k there exists $b(\cdot) \geq 0$ such that

$$v_k(\tilde{\tau}^k(s_k) - b(s_k)) = \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k}) v_k(\bar{\tau}^k(s_k, s^{-k})) \tag{2}$$

with $b(s_k) > 0$ for at least one s_k. If manager k is offered the contract $\tilde{\tau}^k(s_k) - b(s_k)$ for $s_k \in S_k$, independently of s^{-k}, by (2) the participation constraint is still satisfied and, since the coefficient of $p_k(s_k, \bar{e}_k)$ in (1) has not changed, $\bar{e}_k$ is still the optimal effort. However, since $E_{\bar{e}} b(s) > 0$, the expected cost of the contract is lower, contradicting profit maximization. Thus $\bar{\tau}_k(s_k, s^{-k})$ depends only on s_k.

Suppose $(\bar{x}, \bar{\tau}, \bar{e})$ is not CPO. Then there exists an allocation $(\hat{x}, \hat{\tau}, \hat{e})$ such that

$$\sum_{i \in I} \hat{x}^i_s + \sum_{k \in K} \hat{\tau}^k_s = \sum_{k \in K} y^k_s, \quad s \in S \tag{3}$$

$\hat{e}_k$ is optimal for manager k given $\hat{\tau}_k$ and

$$E_{\hat{e}}(\hat{x}^i) \geq E_{\bar{e}}(\bar{x}^i),\ i \in I, \quad E_{\hat{e}}(v_k(\hat{\tau}_k)) - c_k(\hat{e}_k) \geq E_{\bar{e}}(v_k(\bar{\tau}_k)) - c_k(\bar{e}_k),\ k \in K \tag{4}$$

with strict inequality for some i or some k. By the same reasoning as above we know that there exists a contract $\tilde{\tau}^k$, which depends only on s_k such that $\hat{e}_k$ is optimal for this contract and

$$E_{\hat{e}} v(\tilde{\tau}^k) = E_{\hat{e}} v(\hat{\tau}^k), \quad \tilde{\tau}^k \leq \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \hat{e}^{-k})(\hat{\tau}^k(s_k, s^{-k}))$$

Since $(\tilde{\tau}^k, \hat{e}_k)$ satisfy the (PC_k) and (IC_k) constraints, and since $\tilde{\tau}^k$ only depends on s_k, it could have been chosen in the maximization of expected profit. It follows that

$$E_{\bar{e}}(y_k - \bar{\tau}_k) = \sum_{s_k \in S_k} p_k(s_k, \bar{e}_k)(y_k - \bar{\tau}^k(s_k) \geq \sum_{s_k \in S_k} p_k(s_k, \hat{e}_k)(y_k - \tilde{\tau}^k(s_k) \geq E_{\hat{e}}(y_k - \hat{\tau}_k) \quad (5)$$

Suppose that in (4), it is investor i who is strictly better off, $E_{\hat{e}}(\hat{x}^i) > E_{\bar{e}}(\bar{x}^i)$. Then $\sum_{i \in I} E_{\hat{e}}(\hat{x}^i) > \sum_{i \in I} E_{\bar{e}}(\bar{x}^i) = \sum_{k \in K} E_{\hat{e}}(y_k - \bar{\tau}^k) \geq \sum_{k \in K} E_{\hat{e}}(y_k - \hat{\tau}_k)$, which contradicts the feasibility condition (3). Suppose that in (4), it is manager k who is strictly better off with $(\hat{\tau}_k, \hat{e}_k)$. Then the first inequality in (5) must be strict, once again contradicting the feasibility condition (3). For suppose that the first inequality in (5) holds with equality. Since manager k is strictly better off with $(\tilde{\tau}_k, \hat{e}_k)$, the ($\text{PC}_k$) constraint is not binding and $-\infty < v_k(\tilde{\tau}^k)$ implies $\tilde{\tau}^k \gg 0$. Thus for $\epsilon > 0$ sufficiently small and for each state $s_k \in S_k$ the manager's reward can be decreased by $\Delta\tau^k(s_k)$ in such a way that

$$v_k(\tilde{\tau}^k(s_k) - \Delta\tau^k(s_k)) = v_k(\tilde{\tau}^k(s_k)) - \epsilon, \; s_k \in S_k$$

The (PC_k) constraint is still satisfied, and since $E_e(v_k(\tilde{\tau}^k - \Delta\tau^k)) = E_e(v_k(\tilde{\tau}^k)) - \epsilon$ for all e, the optimal effort is still $\hat{e}_k$. But the expected cost can be decreased by $E_{\hat{e}}(\Delta\tau_k)$, which contradicts profit maximization. □

Since with risk neutral investors and independent firms an equilibrium is constrained Pareto optimal, the first-order conditions (i)-(iii) for an equilibrium must coincide with the first-order conditions (i)*-(iii)*, and it is instructive to understand why this is so. (i), (ii) and (i)*, (ii)* clearly coincide, so consider (iii) and (iii)*. Let $p'_k(s_k, \cdot)$ denote the derivative of the function $p_k(s_k, \cdot)$. By the independence assumption

$$\frac{\frac{\partial p_s(e)}{\partial e_k}}{p_s(e)} = \frac{p'_k(s_k, e_k)}{p_k(s_k, e_k)}$$

so that by (ii) the contract of manager k only depends on s_k and not on the realizations of other firms: this property was also derived directly in the proof of Proposition 1 without using the FOCs. The independence assumption also implies that $\dfrac{\partial^2 p_s}{\partial e_k \partial e_j} = \dfrac{\partial p_s}{\partial e_k}\dfrac{\partial p_s}{\partial e_j}/p_s$ so that the second term in (iii)* becomes

$$\sum_{j \neq k} \left(\beta_j \frac{\partial p_s}{\partial e_k} + \mu_j \frac{\partial p_s}{\partial e_k}\frac{\partial p_s}{\partial e_j}/p_s \right) v_j(\tau_s^j) = \sum_{s_k \in S_k} p'_k(s_k, e_k) \sum_{s^{-k} \in S^{-k}} \sum_{j \neq k} (\beta_j + \mu_j \frac{\partial p_s}{\partial e_j}/p_s) v_j(\tau_s^j) p(s^{-k}, e^{-k})$$

which is equal to zero since $\sum_{s_k \in S_k} p'_k(s_k, e_k) = 0$. The third term in (iii)* is zero since the incentive constraint is binding. Since with linear preferences for the investors an interior allocation requires that all the weights of the investors be equal, (iii)* reduces to

$$\sum_{i\in I}\sum_{s\in S} x_s^i \frac{\partial p_s}{\partial e_k} + \mu_k\left(\sum_{s\in S}\frac{\partial^2 p_s}{(\partial e_k)^2}v_k(\tau_s^k) - c''(e_k)\right) = 0, \quad k\in K$$

The feasibility constraint can be written as

$$\sum_{i\in I} x_s^i = \sum_{j\neq k}(y_s^j - \tau_s^j) + (y_s^k - \tau_s^k), \quad s\in S$$

so that

$$\begin{aligned}\sum_{i\in I}\sum_{s\in S} x_s^i \frac{\partial p_s}{\partial e_k} &= \sum_{j\neq k}\sum_{s^{-k}\in S^{-k}}(y_s^j - \tau_s^j)p(s^{-k}, e^{-k})\sum_{s_k\in S_k} p'_k(s_k, e_k) \\ &\qquad + \sum_{s_k\in S_k}(y_s^k - \tau_s^k)p'_k(s_k, e_k)\sum_{s^{-k}\in S^{-k}} p(s^{-k}, e^{-k}) \\ &= \sum_{s_k\in S_k}(y_s^k - \tau_s^k)p'_k(s_k, e_k)\end{aligned} \tag{6}$$

since $\sum_{s_k\in S_k} p'_k(s_k, e_k) = 0$ and $\sum_{s^{-k}\in S^{-k}} p(s^{-k}, e^{-k}) = 1$, and, since risk neutrality implies $\pi_s = 1,\ s\in S$, (6) coincides with the first term of (iii), and (iii)* coincides with (iii).

Since risk neutrality and independence play an essential role in showing the equivalence of (iii) and (iii)*, it seems likely that this equivalence will fail if either risk aversion or independence is not satisfied.

2. Risk Averse Investors and Common Shocks

The analysis of the previous section suggests that a managerial equilibrium will cease to be constrained efficient if investors are risk averse or firms' outcomes are influenced by common shocks. The object of this section is to study the bias in the provision of managerial effort introduced in equilibrium by the presence of risk aversion on the part of the investors or by the presence of common shocks underlying the random outcomes of firms.

Our procedure will be based on a comparison of the first-order conditions $(\text{FOC})_E$ and $(\text{FOC})_{CP}$ at an equilibrium and a constrained Pareto optimum respectively. More precisely the general procedure is as follows. Suppose $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior managerial equilibrium. Under assumptions which will be spelled out below, the first-order approach (replacing the incentive constraints by the first-order condition (IC'_k)) is valid and there exist multipliers $(\bar{\lambda}, \bar{\beta}, \bar{\mu}) = ((\bar{\lambda})_{i\in I}, (\bar{\beta}_k, \bar{\mu}_k)_{k\in K}) \geq 0$ such that (i)-(iii) in $(\text{FOC})_E$ are satisfied. To evaluate the optimality of the equilibrium, consider the social welfare function $W_{\bar{\alpha},\bar{\beta}}(x, \tau, e)$ defined in the previous section where the investors' weights $\bar{\alpha}_i = 1/\bar{\lambda}_i,\ i\in I$, are the inverse of the marginal utilities of income and the managers' weights

$\bar{\beta}_k,\ k \in K$, are the multipliers of the participation constraints (PC$_k$) . Let $RC_s(x,\tau)$ and $IC'_k(\tau,e)$, denote the functions which permit the resource and incentive constraints (RC_s) and (IC'_k) in the previous section to be written as $RC_s(x,\tau) = 0, s \in S$ and $IC'_k(\tau,e) = 0,\ k \in K$. Consider the Lagrangian function $\bar{\mathcal{L}}(x,\tau,e)$ defined by

$$\bar{\mathcal{L}}(x,\tau,e) = W_{\bar{\alpha},\bar{\beta}}(x,\tau,e) - \hat{\pi}RC(x,\tau) + \bar{\mu}IC'(\tau,e)$$

where the multipliers $(\hat{\pi},\bar{\mu})$, with $\hat{\pi}_s = \bar{\pi}_s p_s(\bar{e})$, are evaluated at the equilibrium. With this choice of weights $(\bar{\alpha},\bar{\beta})$ and multipliers $(\hat{\pi},\bar{\mu})$, it is clear that the first-order conditions $(FOC)_E$ (i)-(ii) and (FOC)$_{CP}$ (i*)-(ii*) coincide so that

$$D_x\bar{\mathcal{L}}(\bar{x},\bar{\tau},\bar{e}) = 0, \quad D_\tau\bar{\mathcal{L}}(\bar{x},\bar{\tau},\bar{e}) = 0$$

If we can sign the gradient of $\bar{\mathcal{L}}$ with respect to e, then we can deduce, at least locally, if there is under or over provision of managerial effort at equilibrium.

Proposition 2 *If $(\bar{x},\bar{\tau},\bar{e},\bar{\pi})$ is an interior managerial equilibrium and if $D_e\bar{\mathcal{L}}(\bar{x},\bar{\tau},\bar{e}) \gg 0$, then there exists a constrained feasible marginal reallocation*

$$(\bar{x},\bar{\tau},\bar{e}) \longrightarrow (\bar{x}+\Delta x, \bar{\tau}+\Delta\tau, \bar{e}+\Delta e)$$

with $\Delta e > 0$ which is Pareto improving.

Proof: It is convenient to introduce the following more condensed vector notation: let $p(e) = (p_s(e))_{s\in S},\ u_i(x^i) = (u_i(x^i_s))_{s\in S},\ v_k(\tau^k) = (v_k(\tau^k_s))_{s\in S}$ and for a pair of vectors $x, y \in \mathbb{R}^S$ to let $x \circ y = (x_s y_s)_{s\in S}$ denote the vector in $\mathbb{R}^S$ obtained by component-wise multiplication. Consider any semi-positive[3] marginal change in the vector of effort levels of the managers $\bar{e} \to \bar{e} + \Delta e$ with $\Delta e = (\Delta e_1, \ldots, \Delta e_K) > 0$. Choose a change $\Delta\tau^k$ in the reward of each manager $k \in K$ such that the utility level of the manager is unchanged and the incentive constraint (IC'_k) stays satisfied to terms of first order. Thus for each k we must find $\Delta\tau^k \in \mathbb{R}^S$ such that

$$\begin{aligned} p(\bar{e}) \circ v'_k(\bar{\tau}^k)\Delta\tau^k + D_e p(\bar{e})\Delta e \cdot v_k(\bar{\tau}^k) - c'(\bar{e}_k)\Delta e_k &= 0 \\ D_{e_k}p(\bar{e}) \circ v'_k(\bar{\tau}^k)\Delta\tau^k + D^2_{e,e_k}p(\bar{e})\Delta e \cdot v_k(\bar{\tau}^k) - c''(\bar{e}_k)\Delta e_k &= 0 \end{aligned}$$

The vector $p(\bar{e}) \circ v'_k(\bar{\tau}^k)$ is positive and, since $\sum_{s\in S} \dfrac{\partial p_s}{\partial e_k} = 0$, the vector $D_{e_k}p(\bar{e}) \circ v'_k(\bar{\tau}^k)$ has positive and negative elements. Thus the two vectors are linearly independent, so that a solution $\Delta\tau^k \in \mathbb{R}^S$ to this pair of equations always exists for each $k \in K$.

[3]For $z \in R^K$, z is *semi-positive* (we write $z > 0$) if $z \geq 0$ and $z \neq 0$).

For each investor $i = 2, \ldots, I$ choose a change in consumption $\bar{x}^i \to \bar{x}^i + \Delta x^i$ such that the utility of investor i is unchanged

$$p(\bar{e}) \circ u_i'(\bar{x}^i)\Delta x^i + D_e p(\bar{e})\Delta e \cdot u_i(\bar{x}^i) = 0$$

Finally, for agent 1 choose Δx^1 such that the resource constraints are satisfied, $\sum_{i\in I} \Delta x^i + \sum_{k\in K} \Delta\tau^k = 0$. Let $\bar{\mathcal{L}} = \mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}; \hat{\pi}, \bar{\mu})$; the change in $\mathcal{L}$ induced by the change $(\Delta x, \Delta\tau, \Delta e)$ in the allocation satisfies

$$\Delta\mathcal{L} = D_x\bar{\mathcal{L}}\Delta x + D_\tau\bar{\mathcal{L}}\Delta\tau + D_e\bar{\mathcal{L}}\Delta e > 0$$

since $D_x\bar{\mathcal{L}} = D_\tau\bar{\mathcal{L}} = 0$ and $D_e\bar{\mathcal{L}} \gg 0$. Since $(\Delta x, \Delta\tau, \Delta e)$ has been chosen so that $\Delta RC = 0$, $\Delta IC' = 0$, and the utility of all managers and investors except for investor 1 is unchanged, it follows that $\Delta\mathcal{L} = \Delta W_{\bar{\alpha},\bar{\beta}} = \alpha_1 \Delta(p(\bar{e})u_1(\bar{x}_1)) > 0$, so that the reallocation $(\bar{x}, \bar{\tau}, \bar{e}) \to (\bar{x} + \Delta x, \bar{\tau} + \Delta\tau, \bar{e} + \Delta e)$ is Pareto improving. $\square$

We analyze the effect of removing the assumptions of investor risk neutrality and of independence of firms' outcomes separately. We begin by studying the effect of risk aversion of investors.

2a. Risk Averse Investors

We retain the assumption that firms' outcomes are independent, so that $p(s, e) = \prod_{k=1}^{K} p_k(s_k, e_k)$ and assume that there are not so many firms that the Law of Large Numbers applies, so that even if investors are maximally diversified, there is risk in their equilibrium consumption streams. We study the effect of risk aversion of the investors on the equilibrium outcome. The next proposition shows that under the relatively mild condition that the profit received by investors from each firm is an increasing function of the firm's output, there is under provision of managerial effort at an equilibrium in the sense that there exists a marginal reallocation with higher effort levels for the managers which Pareto improves the equilibrium allocation. To establish this result we make the following assumptions.

A1. The utility functions $(v_k)_{k\in K}$ of managers are differentiable, increasing, strictly concave and $v_k(c) \to -\infty$ as $c \to 0$, for all $k \in K$.

A2. The utility functions $(u_i)_{i\in I}$ of investors are differentiable, increasing, strictly concave and $u_i'(c) \to \infty$ as $c \to 0$, for all $i \in I$.

A3. Firms outcomes are independent.

A4. For all $k \in K$ and $e_k > 0$, $\dfrac{p_k'(s_k, e_k)}{p_k(s_k, e_k)}$ is an increasing function of s_k.

A5. For all $k \in K$, and $\min_{s_k}(y^k_{s_k}) \leq \alpha < \max_{s_k}(y^k_{s_k})$, $1 - F_k(\alpha, e_k) \doteq \sum_{\{s_k | y^k_{s_k} > \alpha\}} p_k(s_k, e_k)$ is a concave, increasing function of e_k.

Proposition 3. *Let A1-A5 be satisfied. If $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior managerial equilibrium such that for all $k \in K$ and all $s^{-k} \in S^{-k}$, $y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})$ is positive and increasing in s_k, then $D_e\bar{\mathcal{L}}(\bar{x}, \bar{\tau}, \bar{e}) \gg 0$.*

Proof. Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be a managerial equilibrium. Assumptions A1, A4, A5 imply that the first-order approach is valid and that the first-order conditions $(\text{FOC})_E$ and $(\text{FOC})_{CP}$ are satisfied at equilibrium and at a CPO respectively.

Since at the equilibrium (iii) of $(\text{FOC})_E$ holds, $D_e\bar{\mathcal{L}} \gg 0$ is equivalent to $A_k(\bar{x}, \bar{\tau}, \bar{e}) > 0$ for all k, where

$$A_k(\bar{x}, \bar{\tau}, \bar{e}) = \frac{\partial \bar{\mathcal{L}}}{\partial e_k} - \bar{\pi} \circ \frac{\partial p(\bar{e})}{\partial e_k} \cdot (y^k - \bar{\tau}^k) - \bar{\mu}_k \left(\frac{\partial^2 p(\bar{e})}{\partial e_k^2} v_k(\bar{\tau}^k) - c''(\bar{e}_k) \right) > 0, \quad k \in K$$

i.e. A_k is obtained by subtracting (iii) from (iii)*. Evaluating $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_k}$ and canceling terms gives

$$A_k(\bar{x}, \bar{\tau}, \bar{e}) = \frac{\partial p(\bar{e})}{\partial e_k} \cdot \left[\sum_{i \in I} \bar{\alpha}_i u_i(\bar{x}^i) + \sum_{j \neq k} \left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p(\bar{e})}{\partial e_j}}{p(\bar{e})} \right) \circ v_j(\bar{\tau}^j) - \bar{\pi} \circ (y^k - \bar{\tau}^k) \right]$$

where we have used the fact that under Assumption A3 of independence $\dfrac{\partial^2 p_s}{\partial e_k \partial e_j} = \dfrac{\partial p_s}{\partial e_k} \dfrac{\frac{\partial p_s}{\partial e_j}}{p_s}$, and where $\dfrac{\frac{\partial p(\bar{e})}{\partial e_j}}{p(\bar{e})}$ denotes the vector of likelihood ratios $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$, $s \in S$. Also note that $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})} = \dfrac{p'_j(s_j, \bar{e}_j)}{p_j(s_j, \bar{e}_j)}$, so that it only varies with s_j.

For $s^{-k} \in S^{-k}$, consider the convolution function $V_{s^{-k}} : \mathbb{R}_{++} \to \mathbb{R}$ defined by

$$V_{s^{-k}}(\xi) = \max \left\{ \sum_{i \in I} \bar{\alpha}_i u_i(\xi_i) + \sum_{j \neq k} \bar{\alpha}_j v_j(\xi_j) \,\middle|\, \sum_{i \in I} \xi_i + \sum_{j \neq k} \xi_j = \xi \right\} \tag{7}$$

with $\bar{\alpha}_j = \bar{\beta}_j + \bar{\mu}_j \frac{p'_j(s_j, \bar{e}_j)}{p_j(s_j, \bar{e}_j)}$. Thus $V_{s^{-k}}$ is the maximized social welfare function formed for all agents except manager k, with managers weighted by their "incentive weights" $\bar{\alpha}_j$. In view of A1 this function is differentiable, increasing and strictly concave. If a vector $(\xi^*_i, i \in I, \xi^*_j, j \neq k)$ is such that $\sum_{i \in I} \xi^*_i + \sum_{j \neq k} \xi^*_j = \xi$ and there exists a vector ρ such that $\bar{\alpha}_i u'_i(\xi^*_i) = \bar{\alpha}_j v'_j(\xi^*_j) = \rho$, then $(\xi^*_i, i \in I, \xi^*_j, j \neq k)$ is a solution to the maximum problem (7), so that $V_{s^{-k}}(\xi) = \sum_{i \in I} \bar{\alpha}_i u_i(\xi^*_i) + \sum_{j \neq k} \bar{\alpha}_j v_j(\xi^*_j)$. In addition, $V'_{s^{-k}}(\xi) = \rho$ (see e.g. Magill-Quinzii (1996, p. 192)).

For any $s^{-k} = (s_j)_{j\neq k} \in S^{-k}$, let $Y(s^{-k}) = \sum_{j\neq k} y^j_{s_j}$ denote the production of all firms excluding k. In state $s = (s_k, s^{-k})$, the investors and the managers other than k share the output $Y(s^{-k}) + y^k_{s_k} - \bar{\tau}^k_s$, and the first-order conditions (i) and (ii) in $(\text{FOC})_E$ imply that

$$V_{s^{-k}}(Y(s^{-k}) + (y^k_{s_k} - \bar{\tau}^k_s)) = \sum_{i\in I} \bar{\alpha}_i u_i(\bar{x}^i_s) + \sum_{j\neq k} \bar{\alpha}_j v_j(\bar{\tau}^j_s)$$

and $V'_{s^{-k}}(Y(s^{-k}) + (y^k_{s_k} - \bar{\tau}^k_s)) = \bar{\pi}_s = \bar{\pi}(s_k, s^{-k})$. Thus $A_k(\bar{x}, \bar{\tau}, \bar{e})$ can be written as

$$\begin{aligned} A_k(\bar{x}, \bar{\tau}, \bar{e}) = \quad & \sum_{s^{-k}\in S^{-k}} p(s^{-k}, \bar{e}^{-k}) \sum_{s_k\in S_k} p'_k(s_k, \bar{e}_k) \Big[V_{s^{-k}}(Y(s^{-k}) + y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) \\ & - V'_{s^{-k}}(Y(s^{-k}) + y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k}))(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) \Big], \quad k \in K \end{aligned}$$

Define $\phi(\chi) = V_{s^{-k}}(Y(s_k) + \chi) - V'_{s^{-k}}(Y(s_k) + \chi)\chi$. Then $\phi'(\chi) = -V''_{s^{-k}}(Y(s_k) + \chi)\chi > 0, \forall \chi > 0$ since $V_{s^{-k}}$ is strictly concave, so that ϕ is an increasing function. The monotone likelihood ratio condition A4 implies that if $\bar{e}_k > \tilde{e}_k$, the distribution function $F(\sigma, \bar{e}_k) = \sum_{s_k\leq\sigma} p_k(s_k, \bar{e}_k)$ first-order stochastically dominates $F(\sigma, \tilde{e}_k)$ (see Rogerson (1985)). It follows that if $y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})$ is an increasing function of s_k then

$$\sum_{s_k\in s_k} p_k(s_k, \bar{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) > \sum_{s_k\in s_k} p_k(s_k, \tilde{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k}))$$

and in the limit when $\tilde{e}_k \to \bar{e}_k$, $\sum_{s_k\in s_k} p'_k(s_k, \bar{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) > 0$. Thus $A_k(\bar{x}, \bar{\tau}, \bar{e}) > 0$ and the proof is complete. □

Remark 2. Proposition 3 requires that the payoff to the shareholders be an increasing function of the firm's output (profit). If the model is viewed as a discrete version of the model with continuous outcomes then the condition requires that the slope $d\tau^k/dy^k$ of the reward schedule $t^k(y^k)$ of the manager of firm k be less than 1. This is a condition which is intuitively reasonable and is certainly satisfied in practice for the observed compensation of CEOs. Murphy (1999) studies the compensation of CEOs for a large sample of leading US corporations during the 1990's and in particular examines how CEO compensation increases (on average) when shareholder wealth increase by 1000\$: the maximum reported number is 35\$ or a slope of 0.035. But of course we cannot be sure that the observed compensation schemes are optimal or close to being optimal. For the model studied in this paper it is easy to specify outputs $(y^k_{s_k})$, probability functions $p_k(s_k, e_k)$, preferences u_i and (v_k, c_k), and reservation utility (ν_k) for the managers, so that the resulting equilibrium compensation $(\bar{\tau}^k)$ schedules satisfy this condition: but we have not found simple clear-cut restrictions on the parameters of the model ensuring that it is always true in equilibrium.

Remark 3. Note that Proposition 3 also holds when effort is observable. This can be seen by setting $\mu_k = 0$ and ignoring the incentive constraints. Thus a managerial equilibrium with observable effort is not first-best optimal. The inefficiency established in Proposition 3 does not come from the moral hazard problem: rather it comes from the structural property of the model by which a manager's effort affects the probability of the firm's outcome. In Magill-Quinzii (2002) we studied an alternative modeling of moral hazard in which states of nature are known, but not verifiable by third parties, and managerial effort affects the output produced in each state. In that model an equilibrium is CPO:[4] this comes from the fact that when managerial effort affects quantities, prices give the right signals for choosing the managers' contracts, since both investors and firms choose quantities of income in each state, probabilities being exogenously given. In the current model, firms take into account the fact that probabilities are influenced by effort an take as given the marginal utility of income in each state, while investors (as consumers) take the probabilities as given and choose quantities. Thus there is no objective market signal of the value of changing probability for investors (as consumers).

Remark 4. The key idea to the proof of Proposition 3 is that the planner in determining the optimal effort e_k of manager k takes into account the change in the expected social welfare[5] $V(Y + y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$ arising from the shift in probability across the stream of net outputs $(y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$, while the market evaluates the increment to the expected value of $V'(Y + y^k_{s_k} - \bar{\tau}^k_{s_k})(y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$. Since V is a concave increasing function, $V(Y+\chi) - V'(Y+\chi)\chi$ is increasing for $\chi > 0$, and the function $V(Y+\chi)$ varies more than its "marginal function" $V'(Y+\chi)\chi$, in the sense that

$$V(Y+\chi_2) - V(Y+\chi_1) > V'Y+\chi_1)\chi_1 - V'(Y+\chi_2)\chi_2, \text{ whenever } \quad \chi_2 > \chi_1 \tag{8}$$

Thus the shift in the probabilities arising from an increment to the effort e_k of manager k creates greater gains in the welfare function of the planner than in the equilibrium profit function, so that the effort chosen by the planner is greater than that in the equilibrium. The difference between the planner and the market's evaluation in (8) is shown in Figure 1. $V(Y+\chi_2) - V(Y+\chi_1)$ is the area DCEFG, while $V'(\bar{Y}+\chi_1)\chi_1 - V'(\bar{Y}+\chi_2)\chi_2$ is the area CEFG minus the area ABCD, and

area CEFG$-$area ABCD$<$area CEFG$<$area DCEFG

The error in the market evaluation is ABCGD. As Figure 1b illustrates, the flatter the marginal function $V'(Y+\chi)$, either because Y is large or because agents are less risk averse, the smaller the

[4]In Magill-Quinzii (2002) each owner-manager chooses his own contract by choosing a portfolio of stocks, bonds and options. It can be shown however that the equilibrium is the same if firms' shareholders choose the optimal contract for the manager, given the appropriate participation and incentive constraints.

[5]To simplify we write V rather than V_{s-k} and Y instead of $Y(s_k)$.

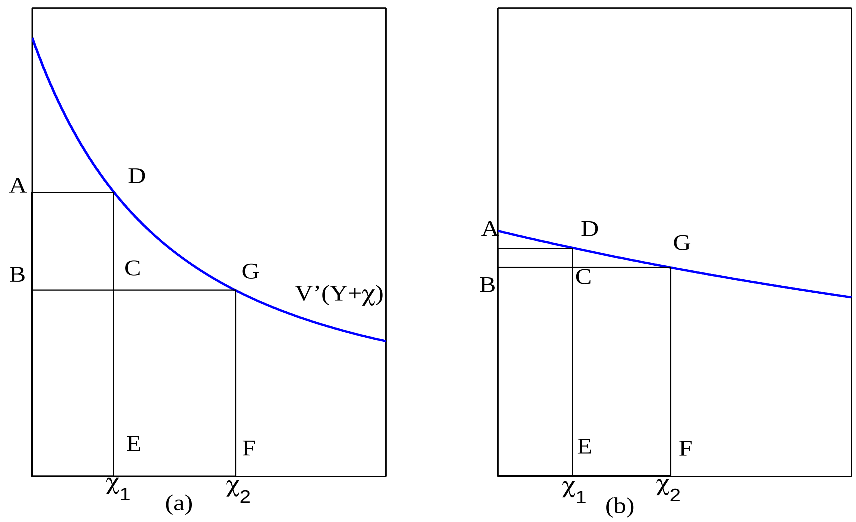

Figure 1: Difference between planner and market evaluation (area ABCGD)

diference between the planner's and the market's evaluation, and hence the smaller the underinvestment in effort at equilibrium.

2b. Common Shocks

In this section we analyze the setting where there is a mutual dependence among firms induced by the presence of a common shock. To clearly understand the effect of such a dependence on the efficiency of the equilibrium we revert to the case of risk neutral investors, so that the inefficiency studied in the previous section disappears.

The common shock is modeled as a random variable η with distribution function $G(\eta)$. We assume that, conditional on the value of η, firms outcomes are independent. For each firm k, let $\rho_k(s_k, e_k, \eta)$ denote the probability of the outcome $y^k_{s_k}$, given the effort level e_k and given a shock η. Then the probability of the joint outcome $s = (s_1, \ldots, s_K)$ given the vector of effort levels $e = (e_1, \ldots, e_K)$ and the shock η is given by

$$\rho(s, e, \eta) = \prod_{k \in K} \rho_k(s_k, e_k, \eta)$$

If the shock η is observable and all the variables are indexed by the shock η, then the analysis of Proposition 1 goes through, and a managerial equilibrium is constrained efficient. We will examine here the case where the shock η is not observable and cannot be deduced with certainty from the observed outcomes of the firms, so that contracts cannot be directly written conditional of the value of η. We assume that investors and managers are symmetrically uninformed about the value of η but know the distribution function G: thus for any agent in the economy the probability of an outcome $y_s = (y^1_{s_1}, \ldots, y^K_{s_K})$ given the effort levels $e = (e_1, \ldots, e_K)$ is given by

$$p(s,e) = \int_{\mathbb{R}} \rho(s,e,\eta) dG(\eta)$$

As usual we will use either the notation $p(s,e)$, or $p_s(e)$, or sometimes just p_s, depending on the complexity of the expression.

In this setting where η is not observable the contract of manager j will depend on the realized outputs of the other firms since these realizations give information on the value of the common shock and, by inference, on the likelihood that the outcome of firm j comes from a high or a low effort of manager j. The dependence of the contract of manager j on the outcome of firm k introduces a dependence of this contract on the effort of manager k, and hence an externality. A (constrained) planner would take this externality into account, while the markets will not. Thus a managerial equilibrium is typically not Pareto optimal. However the sign of the bias is less clear than in the previous section. In Proposition 4 we show that under natural assumptions on the way managerial effort and the common shock interact in the determination of the probability of the outcomes, the derivative of the Lagrangian of the social welfare function with respect to the effort of manager k, evaluated at the equilibrium allocation, is the sum of two terms, one positive, one negative. We use an example to show when the positive or negative term dominates, that is when there is under or over provision of effort at equilibrium.

To analyze the effect of a common shock we make the following assumptions on the characteristics of the economy.

B1. The utility functions $(v_k)_{k\in K}$ of managers are differentiable, increasing, strictly concave, and $v_k(c) \to 0$ as $c \to 0$, for all $k \in K$.

B2. Investors are risk neutral: $u_i(c) = c$, for all $i \in I$.

B3. $p(s,e) = \int_{\mathbb{R}} \prod_{k\in K} \rho_k(s_k, e_k, \eta) dG(\eta)$, for some distribution function G.

B4. For all $k \in K$, $e_k > 0$, and $\eta \in \mathbb{R}$, $\dfrac{\frac{\partial}{\partial e_k}\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is an increasing function of s_k.

B5. For all $k \in K$, $\eta \in \mathbb{R}$, and $\min_{s_k}(y^k_{s_k}) \leq \alpha < \max_{s_k}(y^k_{s_k})$, $\sum_{\{s_k | y^k_{s_k} > \alpha\}} \rho_k(s_k, e_k, \eta)$ is a concave, increasing function of e_k.

B6. For all $k \in K$, $e_k > 0$, and $\eta \in \mathbb{R}$, $\dfrac{\frac{\partial}{\partial \eta}\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is an increasing function of s_k.

B7. For all $k \in K$, $e_k > 0$, and $s_k \in S_k$, $\dfrac{\frac{\partial}{\partial e_k}\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is an decreasing function of η

B3 defines the probability structure: firms' outcomes are affected by the common shock η but, conditional on the value of η, their outcomes are independent random variables. B4 and B5 are the standard properties assumed in the principal-agent model, namely the monotone likelihood ratio property and stochastic decreasing returns to effort, which are assumed to hold for every value of the common shock. B6 is the condition which ensures that a higher value of η is favorable to high outcomes: it is equivalent to the property that, if $\eta > \eta'$, the ratio of the likelihood of $y^k_{s_k}$ with η to the likelihood of $y^k_{s_k}$ with η', namely

$$\frac{\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta')} = \int_{\eta'}^{\eta} \frac{\frac{\partial}{\partial \eta}\rho_k(s_k, e_k, \theta)}{\rho_k(s_k, e_k, \theta)} d\theta$$

increases with s_k. B7 is an assumption on the interaction between the effect of managerial effort and the common shock. It is equivalent to the property that, for $e_k > e'_k$, the likelihood ratio

$$\frac{\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e'_k, \eta)} = \int_{e'_k}^{e_k} \frac{\frac{\partial}{\partial e_k}\rho_k(s_k, t, \eta)}{\rho_k(s_k, t, \eta)} dt$$

decreases with η. The shock and effort are in essence substitutes since increasing η decreases the likelihood that $y^k_{s_k}$ can be attributed to a high rather than a low effort. If η were observable, the compensation of manager k would decrease as η increases. When η is not observable but B6 holds, the outcomes of firms $j \neq k$ give information on the likelihood that η has been high or low, and this leads to a monotone dependence of manager k's compensation on the outcomes of other firms $j \neq k$. We say that manager k's compensation $\tau^k(s_k, s^{-k})$ is decreasing in s^{-k} if for all pairs of outcomes $s^{-k} = (s_j)_{j \neq k}$ and $\tilde{s}^{-k} = (\tilde{s}_j)_{j \neq k}$, with $s_j \geq \tilde{s}_j$ for all $j \neq k$ and at least one strict inequality, $\tau^k(s_k, s^{-k}) < \tau^k(s_k, \tilde{s}^{-k})$.

Lemma 1. *Under the assumptions B1-B7, if $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior managerial equilibrium , then for any $k \in K$ and $s_k \in S_k$, the contract $\bar{\tau}^k(s_k, s^{-k})$ is decreasing in s^{-k}.*

The proof is given in Magill-Quinzii (2004), as well as examples which do and do not satisfy B7. Assumption B7 is satisfied when the probability $\rho_k(s_k, e_k, \eta)$ depends additively on e_k and η.

As we have pointed out, the fact that the compensation of the manager of one firm depends on the outcomes of the other firms, which in turn depend on the effort of their managers, implies that the effort of manager k creates an external effect on the other managers $j \neq k$. This external effect is not in the characteristics of the economy since, conditional on η, the firms' outcomes are independent: rather it comes from the combined non-observability of effort and the common shock, which makes it worthwhile for the shareholders of firm j to extract information by observing the outcomes of the other firms. Since typically externalities are not properly taken into account in a market equilibrium we should expect that $\frac{\partial \bar{\mathcal{L}}}{\partial e_k}$ is non zero. In the next propositon we show that $\frac{\partial \bar{\mathcal{L}}}{\partial e_k}$ can be decomposed into the sum of two terms, the *direct effect* D_k, and the *information effect* I_k. The direct effect is the most intuitive: since the payment of manager j decreases with the outcome of firm k, increasing the effort e_k increases the probability of high values of s_k, and this decreases the welfare of manager j. The information effect is more subtle and has its origin in the fact that the likelihood ratio of one manager depends on the effort of the other managers, so that a change in the effort e_k changes the information on the effort of manager j that can be deduced by observing firm j 's outcome. We will come back to the interpretation of the terms D_k and I_k which are defined in Proposition 4 after the proof. To sign these terms we need the assumption that the managers' utility levels are sufficiently high: this is without loss of generality since the analysis is invariant to adding constants to the utility functions $(v_k)_{k \in K}$ (and of course to the guaranteed utility levels ν_k).

Proposition 4. *Let B1-B7 be satisfied. If $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior managerial equilibrium, then, for all $k \in K$, $D_{e_k}\mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}) = D_k + I_k$, where*

$$\begin{aligned} D_k &= \sum_{j \neq k} \sum_{s \in S} \bar{\alpha}_s^j \left(v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j \right) \frac{\partial p_s(\bar{e})}{\partial e_k} \\ I_k &= \sum_{j \neq k} \sum_{s \in S} \bar{\mu}_j p_s(\bar{e}) \frac{\partial}{\partial e_k} \left(\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})} \right) v_j(\bar{\tau}_s^j) \end{aligned}$$

where $\bar{\alpha}_s^j = \bar{\beta}_j + \bar{\mu}_j \dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$. If the utility functions $(v_k)_{k \in K}$ are such that

$$v_k(\bar{\tau}_s^k) - v_k'(\bar{\tau}_s^k)\bar{\tau}_s^k > 0, \quad \forall\, s \in S, \quad \forall\, k \in K \tag{9}$$

then $D_k < 0$ and $I_k > 0$.

Proof: Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be an interior managerial equilibrium and let $(\bar{\lambda}, \bar{\beta}, \bar{\mu})$ be the associated multipliers for which (FOC)$_E$ hold. Since investors are risk neutral we can assume that $\bar{\pi}_s = 1$ for all $s \in S$ and $\bar{\alpha}_i = \frac{1}{\lambda_i} = 1$ for all $i \in I$. Since at the equilibrium the FOC for optimal effort, (iii) of

$(\mathrm{FOC})_E$ is satisfied for each firm it follows that

$$D_{e_k}\mathcal{L}(\bar{x},\bar{\tau},\bar{e}) = \sum_{i\in I}\sum_{s\in S}\bar{x}_s^i\frac{\partial p_s(\bar{e})}{\partial e_k} + \sum_{j\neq k}\sum_{s\in S}\left(\bar{\beta}_j\frac{\partial p_s(\bar{e})}{\partial e_k} + \bar{\mu}_j\frac{\partial^2 p_s(\bar{e})}{\partial e_j \partial e_k}\right)v_j(\bar{\tau}_s^j) - \sum_{s\in S}(y_s^k - \bar{\tau}_s^k)\frac{\partial p_s(\bar{e})}{\partial e_k}$$

From the market clearing conditions, it follows that $\sum_{i\in I}\bar{x}_s^i - (y_s^k - \bar{\tau}_s^k) = \sum_{j\neq k}(y_s^j - \bar{\tau}_s^j)$, for all $s\in S$. Let us show that $\sum_{s\in S} y_s^j \dfrac{\partial p_s(\bar{e})}{\partial e_k} = 0$, for each $j\neq k$. Using the notation $\rho^{-k}(s^{-k},\bar{e}^{-k},\eta) = \prod_{j\neq k}\rho_j(s_j,\bar{e}_j,\eta)$

$$\sum_{s\in S} y_s^j\frac{\partial p_s(\bar{e})}{\partial e_k} = \int_{\mathbb{R}}\sum_{s^{-k}\in S^{-k}}\rho^{-k}(s^{-k},\bar{e}^{-k},\eta)y_{s_j}^j\left(\sum_{s_k\in S_k}\frac{\partial \rho_k(s_k,\bar{e}_k,\eta)}{\partial e_k}\right)dG(\eta) = 0$$

since $\displaystyle\sum_{s_k\in S_k}\frac{\partial \rho_k(s_k,\bar{e}_k,\eta)}{\partial e_k} = 0$. Thus

$$D_{e_k}\mathcal{L}(\bar{x},\bar{\tau},\bar{e}) = \sum_{j\neq k}\sum_{s\in S}\left(\bar{\beta}_j\frac{\partial p_s(\bar{e})}{\partial e_k} + \bar{\mu}_j\frac{\partial^2 p_s(\bar{e})}{\partial e_j\partial e_k}\right)v_j(\bar{\tau}_s^j) - \bar{\tau}_s^j\frac{\partial p_s(\bar{e})}{\partial e_k} \tag{10}$$

Adding and subtracting the terms $\bar{\mu}_j\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})}v_j(\bar{\tau}_s^j)$ and using equation (ii) in $(\mathrm{FOC})_E$ with $\bar{\pi}_s = 1$, gives the decomposition

$$D_{e_k}\mathcal{L}(\bar{x},\bar{\tau},\bar{e}) = D_k + I_k, \quad D_k = \sum_{j\neq k} D_{j,k}, \quad I_k = \sum_{j\neq k} I_{j,k}$$

with

$$D_{j,k} = \sum_{s\in S}\left(\bar{\beta}_j + \bar{\mu}_j\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right)\left(v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j\right)\frac{\partial p_s(\bar{e})}{\partial e_k}$$

$$I_{j,k} = \sum_{s\in S}\bar{\mu}_j\left(\frac{\partial^2 p_s(\bar{e})}{\partial e_j\partial e_k} - \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})}\right)v_j(\bar{\tau}_s^j)$$

Note that $\dfrac{\partial}{\partial e_k}\left(\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) = \dfrac{\frac{\partial^2 p_s(\bar{e})}{\partial e_j\partial e_k}p_s(\bar{e}) - \frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})^2}$, so that $I_{j,k}$ can also be written as

$$I_{j,k} = \sum_{s\in S}\bar{\mu}_j p_s(\bar{e})\frac{\partial}{\partial e_k}\left(\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right)v_j(\bar{\tau}_s^j)$$

Sign of $\boldsymbol{D_{j,k}}$: Since $a_j v_j + b_j$ for $a_j > 0$ represents the same preferences for manager j as v_j and since the consumption vector $\bar{\tau}_s^j$ is bounded, we can assume without loss of generality that

b_j is chosen such that (9) holds. As we saw in the proof of Proposition 3, $x \to v_j(x) - v'(x)x$ is an increasing function of x. Since by Lemma 1 $\tau^j(s_k, s^{-k})$ is decreasing in s_k, the function $v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j$ is decreasing in s_k. Since $1 = \left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) v_j'(\bar{\tau}_s^j)$, and v_j' is decreasing, $\bar{\tau}_s^j$ decreasing in s_k is equivalent to $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$ decreasing in s_k. Thus the product

$$H_j(s_k, s^{-k}) = \left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) \left(v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j\right)$$

is a decreasing function of s_k as a product of positive decreasing functions of s_k, and $D_{j,k}$ can be written as

$$D_{j,k} = \int_{\mathbb{R}} \sum_{s^{-k} \in S^{-k}} \rho^{-k}(s^{-k}, \bar{e}^{-k}, \eta) \sum_{s_k \in S_k} H_j(s_k, s^{-k}) \frac{\partial \rho(s_k, \bar{e}_k, \eta)}{\partial e_k} dG(\eta)$$

The monotone likelihood ratio implies that if $e_k > e_k'$ the distribution function generated by $\rho(s_k, e_k, \eta)$ first-order stochastically dominates the distribution function generated by $\rho(s_k, e_k', \eta)$, which implies that $\sum_{s_k \in S_k} H_j(s_k, s^{-k}) \dfrac{\partial \rho(s_k, \bar{e}_k, \eta)}{\partial e_k} < 0$ since $H_j(s_k, s^{-k})$ is decreasing in s_k. Thus $D_{j,k} < 0$.

Sign of $\boldsymbol{I_{j,k}}$: Let us show that $\dfrac{\partial^2 p_s(e)}{\partial e_j \partial e_k} > \dfrac{\frac{\partial p_s(e)}{\partial e_j} \frac{\partial p_s(e)}{\partial e_k}}{p_s(e)}$, for all $s \in S$ and all $e \gg 0$. Since $v_j(\bar{\tau}_s^j) > v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j > 0$, this will imply that $I_{j,k} > 0$. Note that

$$\frac{1}{p_s(e)} \frac{\partial^2 p_s(e)}{\partial e_j \partial e_k} = \int_{\mathbb{R}} L_j(s_j, e_j, \eta) L_k(s_k, e_k, \eta) a(s, e, \eta) dG(\eta) \tag{11}$$

where $L_k(s_k, e_k, \eta) = \dfrac{\frac{\partial}{\partial e_k} \rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is the local likelihood function of manager k and where $a(s, e, \eta) = \dfrac{\rho(s, e, \eta)}{\int_{\mathbb{R}} \rho(s, e, \eta) dG(\eta)}$ is a density function for the measure $dG(\eta)$. Let G_a denote the distribution function induced by the density a with respect to dG. The integral (11) is the expectation of the product of the random variables L_j and L_k with respect to dG_a so that

$$\frac{1}{p_s(e)} \frac{\partial^2 p_s(e)}{\partial e_j \partial e_k} = E_a(L_j L_k) = E_a(L_j) E_a(L_k) + \mathrm{cov}_a(L_j, L_k) = \frac{\partial p_s(e)}{\partial e_j} \frac{\partial p_s(e)}{\partial e_k} + \mathrm{cov}_a(L_j, L_k)$$

Thus the sign of the difference $\dfrac{\partial^2 p_s(e)}{\partial e_j \partial e_k} - \dfrac{\frac{\partial p_s(e)}{\partial e_j} \frac{\partial p_s(e)}{\partial e_k}}{p_s(e)}$ is the sign of the covariance term. By B7 the random variables L_j and L_k are decreasing functions of η, and are thus positively dependent random variables with respect to dG_a. This in turn implies that $\text{cov}_a(L_j, L_k)$ is positive (see e.g. Magill-Quinzii (1996, p.170)). $\square$

The general principle underlying an incentive contract is that the agent undertaking the effort should be paid more when the realized outcome is more likely to have occurred with high effort, and should be paid less when the outcome is more likely with low effort. When outcomes are the combined result of effort and a common shock—and when the shock is not observable but also affects other firms—then the realized outcomes of these other firms provide information on the shock, and this in turn provides information on the likelihood that a given outcome for the firm is due to high or low effort on the part of its manager. Since the outcomes of other firms are also influenced by the effort of their managers, the fact that observed outcomes are used to infer information about the unobservable common shock introduces a dependence between the effort of manager k and the compensation of manager $j \neq k$. The contract of manager k in equilibrium only takes into account the effect of his effort on the expected profit of the firm and his expected utility, but ignores its effect on the compensation, and hence the expected utility, of the managers of the other firms. Proposition 4 can be interpreted as a description of the additional effects that a planner would take into account when deciding on the effort to induce from manager k.

The first— the direct effect—would take into account the effect of e_k on the expected utility of manager j, where this utility is evaluated using the state-dependent weights $\bar{\alpha}_s^j = \bar{\beta}_j + \bar{\mu}_j \dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$, $s \in S$, which are used to calculate the optimal risk for manager j given the need to induce appropriate effort on his part. Given the assumption of a decreasing likelihood ratio with respect to the common shock, manager j is paid less when the outcome of firm k is higher: thus decreasing e_k would increase the expected utility of manager j.

The second effect that the planner would take into account is that the effort e_k of manager k influences the local likelihood ratios $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$, and hence the incentive-corrected weights of manager j in the Lagrangian of the social welfare problem. Given the assumption on the way effort and the common shock influence the likelihood of the outcomes—in particular Assumption B7 which in essence implies that managerial effort and the common shock are substitutes in the creation of good outcomes—increasing e_k decreases the estimate of η from the observation of $y^k_{s_k}$, which in turn increases the estimate of e_j which can be inferred from a given realization $y^j_{s_j}$ of firm j. Since this effect occurs through the likelihood ratio, or the information that can be inferred from a given realization of firm j, we call it the information effect.

Example. The following example, which satisfies Assumptions B1-B7, is instructive for studying which of the two effects dominates, i.e. whether there is under or over provision of effort at equilibrium. Let $K = 2$, $S_1 = \{g_1, b_1\}$, $S_2 = \{g_2, b_2\}$, $S = S_1 \times S_2$, $v_k(c) = \dfrac{1}{1-\alpha}c^{1-\alpha}$, $0 < \alpha \neq 1$, and let the probabilities be given by

$$\rho_k(g_k, e_k, \eta) = a_k + b_k e_k + d\eta, \quad 0 < a_k + b_k + d < 1, \quad \rho_k(b_k, e_k, \eta) = 1 - \rho_k(g_k, e_k, \eta), \quad k = 1, 2$$

where η is uniformly distributed on $[0, 1]$ and the cost functions $c_1(e_1)$, and $c_2(e_2)$ are such that e_1 and e_2 always lie in $(0, 1)$, i.e. $c_k(0) = 0$, $c_k(e_k) \to \infty$ as $e_k \to 1$.

To compute an equilibrium we need in addition to specify the outputs $y^k = (y^k_{g_k}, y^k_{b_k})$ of the two firms ($k = 1, 2$), the outside options (ν_1, ν_2) and the cost functions (c_1, c_2) of the two managers. However since the expression (10) that we want to study only depends indirectly on these characteristics through the resulting equilibrium values $(\bar{e}_k, \bar{\beta}_k, \bar{\mu}_k)$, $k = 1, 2$, it is more convenient to study (10) by treating the equilibrium values as parameters. For once $(a_k, b_k, d, \bar{e}_k, \bar{\beta}_k, \bar{\mu}_k)$, $k = 1, 2$ have been chosen, there exist characteristics (y^k, ν_k, c_k) , $k = 1, 2$, consumption streams, contracts and prices $(\bar{x}, \bar{\tau}, \bar{\pi})$ such that $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an equilibrium. Clearly $\bar{\pi}_s = 1$, $\forall s \in S$, and $\bar{\tau}^k$ is such that

$$\bar{\tau}^k_s = \left(\bar{\beta}_k + \bar{\mu}_k \frac{\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})} \right)^{\frac{1}{\alpha}}$$

where $p_s(\bar{e}) = \int_0^1 \rho_1(s_1, \bar{e}_1, \eta)\rho_2(s_2, \bar{e}_2, \eta)d\eta$. Calculating $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$, $j = 1, 2$, and varying the parameters $(\alpha, a, b, d,)(\bar{e}, \bar{\beta}, \bar{\mu})$, we find that the typical graph of $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ as a function of d—which parameterizes the magnitude of the impact of the shock η on the probability of the outcomes of each firm—has the form shown in Figure[6] 2.

When there is no common shock ($d = 0$) the equilibrium is efficient. For small magnitudes of d, the direct externality effect dominates and $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ is negative: managers over invest in effort. When d is sufficiently large, the information effect—which, as we saw in the proof of Proposition 4, is a positive covariance term between two random variables jointly influenced by η—becomes strong enough to dominate. To the extent that in practice the outcomes (profits) of firms are quite strongly correlated, it seems natural within the framework of this model to adopt a relatively large value of d, so that the latter scenario seems more likely. Since, as we saw in Proposition 3, investors' risk aversion also makes $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ positive, the effect of risk aversion combined with that of a common

[6] Figure 2 has been obtained using the following values: $a = (0.25, 0.25)$, $b = (0.2, 0.2)$, $\alpha = 0.5$, $\bar{e} = (0.2, 0.2)$, $\bar{\beta}_1 = 100$, $\bar{\mu}_1 = 50$.

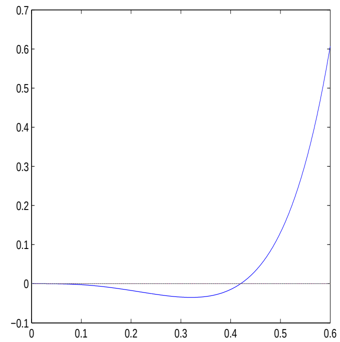

Figure 2: $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_2}$ as a function of d, which parameterizes the impact of the common shock η on the probabilities.

unobservable shock seems likely to lead to under provision of effort in equilibrium, in the sense of Proposition 2.

2c. Continuum of Firms.

Many of the models which study moral hazard in a general equilibrium framework are motivated by the problem of moral hazard in insurance, and make the assumption that there is a continuum of agents of each type, a natural assumption in the context of insurance (Prescott-Townsend (1984a), (1984b), Kocherlakota (1998), Lisboa (2001)). The papers just cited reach the conclusion that an equilibrium is CPO, while we reach a different conclusion. Thus it is instructive to see what happens in our model if we replicate the firms and, in the limit, have a continuum of firms of each type. We will not write out the details of the model for the continuum case, but rather indicate, using the structure of our model, why the inefficiencies studied in Sections 2a and 2b disappear when there is a continuum of firms of each type.

Consider first the model of Section 2a and let us change the model by assuming that $k \in K$ represents a type of firm and that there is a continuum of mass 1 of identical firms of each type. We assume that the probabilities of the outcomes of any two firms (whether of the same or of different types) are independent, and that firms of the same type k have identical managers (same (v_k, ν_k, p_k)). Assuming that all the managers of the same type are offered the same contract and choose the same effort, in equilibrium as well as in the planner's problem, the probabilities $p_k(s_k, e_k)$, $s_k \in S_k$ of the outcomes of firms of type k become the proportion of firms of this type with output s_k, so that the total output $\sum_{s_k} p_k(s_k, e_k) y^k_{s_k}$ of the firms of type k is non-random, and increases with e_k. The continuum of firms eliminates risk and thus the effect of risk aversion studied in Section 2a. Or, another way of looking at it, the trade-off between the cost of providing incentive and the probability of good outcomes faced by an individual firm becomes, at the aggregate level, a trade-off between quantity of output and cost of incentives, and the marginal value of more output is well taken into account in the market.

For the model of Section 2b with a common shock, satisfying the assumptions B1-B7, consider adding a continuum of firms of each type $k \in K$, assuming that the probabilities of the outcomes of any two firms are independent conditional on the value of η. The continuum removes the idiosyncratic shocks of firms from the aggregate: since the optimal effort e_k of a representative manager can be deduced from the incentive contract of firms of type k, and since the proportion of the firms with output s_k can be observed, the probabilities $\rho(s_k, e_k, \eta)$ can be inferred, and from this the value of η can be deduced. Thus the continuum in essence transforms the unobservable η into an observable or inferrable η, and this solves the information problem without introducing an externality. Given Assumption B6 which implies that if $\eta > \eta'$ the distribution function induced by $\rho_k(s_k, e_k, \eta)_{s_k \in S_k}$ first-order stochastically dominates the distribution function for $\rho_k(s_k, e_k, \eta')_{s_k \in S_k}$, the total output $\sum_{s_k} \rho_k(s_k, e_k, \eta) y^k_{s_k}$ of the firms of type k is an increasing function of η. Thus the optimal contract for the representative manager of a type k firm when η is known can equivalently be expressed as a contract which depends on the total output of the firms of type k or the economy-wide aggregate output. Thus even if there is a common shock, if there is a continuum of firms of each type and investors are risk neutral, a managerial equilibrium is constrained Pareto optimal.

In Sections 2a and 2b we have separated the effect of risk aversion and the informational problem induced by the unobservability of the common shock. In the case where there is a common shock and investors are risk averse, constrained Pareto optimality will be obtained with a continuum of firms if there are appropriate markets which permit the aggregate risk induced by η to be optimally shared.

References

Holmström, B. (1982), "Moral Hazard in Teams", *Bell Journal of Economics*, 13, 324-340.

Jensen, M.C. and K.J. Murphy (1990), "Performance Pay and Top-Management Incentives", *Jounal of Political Economy*, 98, 225-264.

Jewitt, I., "Justifying the First-Order Approach to Principal-Agent Problems", *Econometrica*, 1988, 56, 1177-1190.

Kocherlakota, N.R., "The Effect of Moral Hazard on Asset Prices When Financial Markets are Complete", *Journal of Monetary Economics*, 1998, 41, pp. 39-56.

Lisboa, M.B., "Moral Hazard and General Equilibrium in Large Economies", *Economic Theory*, 2001, 18, 555-575.

Magill, M and M. Quinzii (2002), "Capital Market Equilibrium with Moral Hazard", *Journal of Mathematical Economics,* 38, 149-190.

Magill, M and M. Quinzii (2004), "Common Shocks and Relative Compensation Schemes", Working Paper, University of Southern California.

Milgrom, P. (1981), "Good News and Bad News: Representation Theorems and Applications", *Rand Journal of Economics*, 12, 380-391.

Mookherjee, D. (1984), "Optimal Incentive Schemes with Many Agents", *Review of Economic Studies*, 51, 433-446.

Murphy, K.J.(1999), "Executive Compensation", *Handbook of Labor Economics, Volume 3B*, O. Ashenfelter and D. Card eds, North-Holland.

Prescott, E.C. and R.M. Townsend (1984a), "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", *Econometrica*, 52, pp. 21-46.

Prescott, E.C. and R.M. Townsend (1984b), "General Competitive Analysis in an Economy with Private Information", *International Economic Review*, 25, pp. 1-20.

Rogerson, W.P., “The First Order Approach to Principal-Agent Problems”, *Econometrica*, 1985, 53, 1357-1367.